2 Digital Filters and Subband Synthesis

The PCM output of an MP3 decoder is computed from so called subbands. Each subband, there are 32 of them, contains the audible sound limited to a very narrow part of the frequency spectrum. Think of it as a kind of equalizer with 32 sliders; the first slider is for the lowest sounds and the last slider is for the highest tones. If we turn down all the sliders except one, we will hear only part of the music, namely the musical signal limited to a small part of the audible spectrum. This limited part of the spectrum is a subband. We will be able to hear the music, as it was meant to be, only if we open all sliders of our equalizer. This will combine all the signals of the different subbands and will yield the full spectrum.

The musical signal is split into subbands by a digital filter in the encoder and needs to be recombined in the decoder. To understand this process, we consider a simple example: We take a sine wave as input signal, were the amplitude at time t is given by the formula $f(t) = \sin(2\pi t)$. The signal is periodic with period length 1, i.e. the signal between 0 and 1 is repeated between 1 and 2 and so forth. This signal is periodically sampled with period length 1/10, i.e. at $t = 0.0, 0.1, \ldots, 1.0$, which yields the values $x_i = f(i/10)$, that is $x_0 = 0.0$, $x_1 = 0.59$, $x_2 = 0.95$, $x_3 = 0.98$, $x_4 = 0.59$, $x_5 = 0.0$, $x_6 = -0.59$, $x_7 = -0.95$, $x_8 = -0.98$, $x_9 = -0.59$, and $x_{10} = x_0 = 0.0$, as shown in Fig. 1.

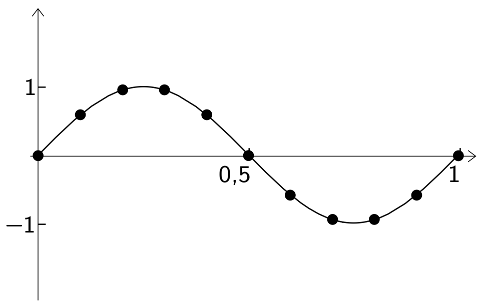


Fig. 1: The filter input

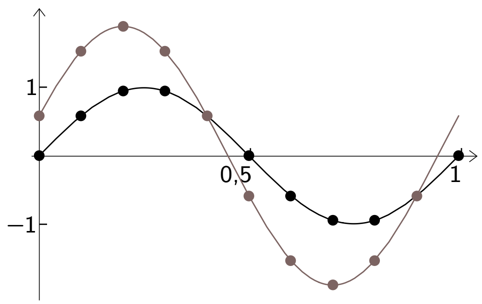


Fig. 2: The filter input and output

2.1 A Simple Filter

Now we examine the most simple digital filter: The output y_i of the filter is computed by $y_i = x_i + x_{i+1}$ from two successive input values. The filters output is shown in Fig. 2 together with its input.

It is immediately visible, that the output is again a sine wave of the same frequency but with a different gain (amplitude) and slightly shifted to the left (phase shift). Employing some useful 9th grade math, or equivalently a tool like Mathematica[28], one can show that in general

$$\sin(\omega t) + \sin(\omega(t + \delta)) = 2\cos(\omega\delta/2)\sin(\omega t + \omega\delta/2).$$

With $\omega = 2\pi$ and $\delta = 1/10$, in our example $\sin(\omega t))$ corresponds to x_i and $\sin(\omega(t+\delta)$ corresponds to x_{i+1}, and we have $y_i = x_i + x_{i+1} = 2\cos(\omega\delta/2)\sin(\omega t + \omega\delta/2)$.

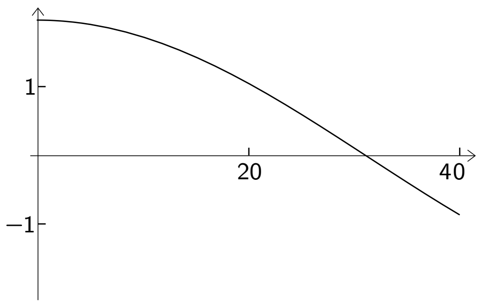


Fig. 3: The filters frequency response

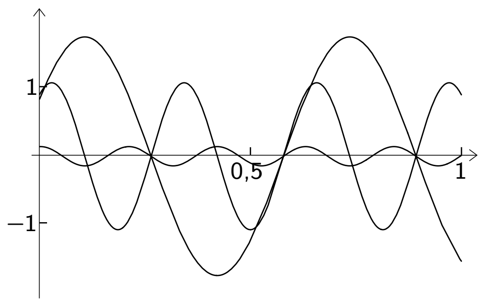


Fig. 4: Filter outputs for unit-gain inputs

The $\sin(\omega t + \omega\delta/2)$ part shows that the filter output is indeed a sine wave with the same frequency (given by ω) but shifted by $\omega\delta/2$ to the left. The factor $2\cos(\omega\delta/2)$, which does not depend on t, is the amplitude of the filter output. It depends on δ and ω. With a fixed sample rate, given by $\delta = 1/10$, the new amplitude depends only on the input frequency ω.

Plotting the filters gain $2\cos(\omega\delta/2)$ against the frequency of the filter input, we obtain Fig. 3, called the filters frequency response. It shows that the filter amplifies low frequencies by almost a factor of two and mutes input signals with higher frequencies (ω close to 30). Fig. 4 illustrates this effect. It shows the filter output for the three inputs $\sin(10t)$, $\sin(20t)$, and $\sin(30t)$.

2.2 The Complementary Filter

Next, we consider the complementary digital filter with output y'_i given by the formula $y'_i = -x_i + x_{i+1}$. Its filter gain is computed as before and turns out to be $2\sin(\omega\delta/2)$. Fig. 6 shows the frequency response of both filters.

It is clearly visible how it suppresses lower frequencies and boosts higher frequencies. Both filters together can now be used to separate any input signal into two subbands,

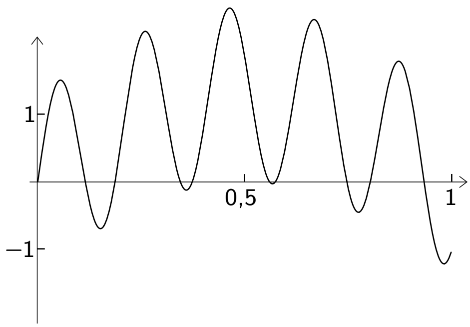
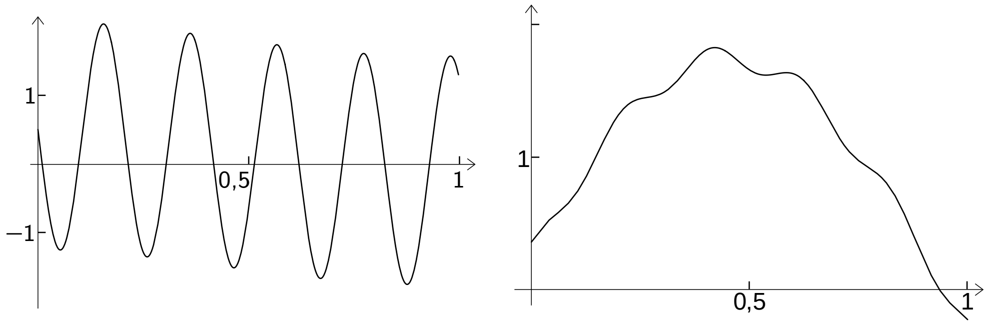


Fig. 5: The input signal to the left was created by adding a low frequency component and a high frequency component

$$\sin(1\pi t) + \sin(5\pi t).$$

This signal was then filtered by the simple low frequency filter and its complementary high frequency filter. The results are shown below.



one containing high frequency content and the other low frequency content. The input function $\sin(1\pi t)+\sin(5\pi t)$ is a good example (see Fig. 5). While the separation of low and high frequencies in the output is clearly visible, it can also be seen that there is a wide overlap of both filters: the low frequency output still contains some significant high frequency content and vice versa.

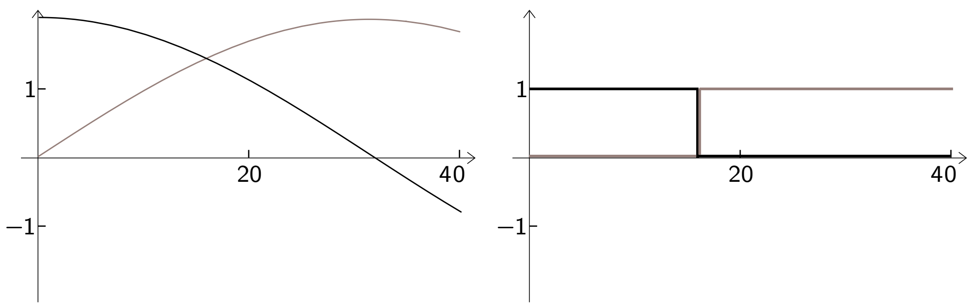


Fig. 6: Frequency response of both filters *Fig. 7: Perfect frequency response*

Of course, much better filters can be constructed, as we will see, but for now, we stick to the simple filters and try to find out, how the original signal can be restored from the output of the two filters. This is the same problem faced by the decoder, where the output must be computed from the subbands.